

1)

a) Answer: $I_e = 300 \frac{W}{m^2}$

The irradiance of the solar simulator is calculated by integrating the spectral irradiance over wavelength:

$$I_e = \int_0^\lambda I_{e\lambda} d\lambda = \int_{300nm}^{500nm} (7,5 \cdot 10^{15} \lambda - 2,25 \cdot 10^3) d\lambda + \int_{500nm}^{1500nm} (-1,5 \cdot 10^{15} \lambda + 2,25 \cdot 10^3) d\lambda$$

$$= 7,5 \cdot 10^{15} \cdot \frac{(500 \cdot 10^{-9})^2 - (300 \cdot 10^{-9})^2}{2} - 2,25 \cdot 10^3 \cdot (500 \cdot 10^{-9} - 300 \cdot 10^{-9}) +$$

$$2,25 \cdot 10^3 \cdot (1500 \cdot 10^{-9} - 500 \cdot 10^{-9}) - 1,5 \cdot 10^{15} \cdot \frac{(1500 \cdot 10^{-9})^2 - (500 \cdot 10^{-9})^2}{2} = 300 \frac{W}{m^2}$$

Alternatively (and much easier), we can just calculate the area under the red triangle:

$$I_e = \frac{200 \cdot 10^3 \cdot 1,5 \cdot 10^3}{2} + \frac{1000 \cdot 10^3 \cdot 1,5 \cdot 10^3}{2} = 300 \frac{W}{m^2}$$

b) Answer: $\Phi_{ph} = 3,48 \cdot 10^{21} \frac{1}{s m^2}$

Applying the given formula for photon flux:

$$\Phi_{ph} = \int_0^\lambda \Phi_{ph} d\lambda = \int_0^\lambda I_{e\lambda} \frac{1}{h\nu} d\lambda$$

$$I = \Phi \cdot E_{photon} = \Phi \cdot \frac{hc}{\lambda}$$

$$\Phi_{ph} = \frac{1}{hc} \left[\int_{300nm}^{500nm} (7,5 \cdot 10^{15} \lambda^2 - 2,25 \cdot 10^3 \lambda) d\lambda + \int_{500nm}^{1500nm} (2,25 \cdot 10^3 \lambda - 1,5 \cdot 10^{15} \lambda^2) d\lambda \right]$$

$$= \frac{1}{hc} \left[\left[7,5 \cdot 10^{15} \frac{\lambda^3}{3} - 2,25 \cdot 10^3 \frac{\lambda^2}{2} \right]_{300nm}^{500nm} + \left[2,25 \cdot 10^3 \frac{\lambda^2}{2} - 1,5 \cdot 10^{15} \frac{\lambda^3}{3} \right]_{500nm}^{1500nm} \right]$$

$$= 0,33 \cdot 10^{21} \frac{1}{s m^2} + 3,15 \cdot 10^{21} \frac{1}{s m^2} = 3,48 \cdot 10^{21} \frac{1}{s m^2}$$

c) Answer: Junction A

Junction A has the ^{higher} band gap of the two junctions. Since the more energetic photons have smaller penetration depth in materials, the high band gap junction should always act as the top cell. Similarly, less energetic photons (near infrared light) have larger penetration depths. Therefore the bottom cell should be the cell with the lowest band gap. The bottom cell will then collect the photons that have not been collected by the top cell since their energy was not larger than the band gap of cell A.

d) Answer: $E_g = 1,77 eV$

The energy of a photon with wavelength λ in eV is given by: $E_{ph} = \frac{hc}{\lambda q}$

Using this formula for wavelength $\lambda = 700nm$ will give:

$$E_g = \frac{6,626 \cdot 10^{-34} J \cdot 2,998 \cdot 10^8 \frac{m}{s}}{700 nm \cdot 1,602 \cdot 10^{-19} C} = 1,77 eV$$

Note that you have to use the highest wavelength of junction A since this will give the lowest energy needed to excite electrons (the band gap energy).



1)

c) Answer: $J_{sc} = 14,6 \text{ mA/cm}^2$

The ~~shorted~~ short circuit current density can be calculated as:

$$J_{sc} = q \int EQE(\lambda) \Phi_{ph,\lambda} d\lambda = q \int EQE(\lambda) \frac{hc}{e\lambda} d\lambda$$

The EQE of junction A is 0,8 for $300 \text{ nm} < \lambda < 700 \text{ nm}$, which can be seen from the graph.

This gives:
$$J_{sc} = \frac{q}{hc} \cdot 0,8 \int_{300 \text{ nm}}^{700 \text{ nm}} \frac{hc}{\lambda} d\lambda = \frac{0,8q}{hc} \left[\int_{300 \text{ nm}}^{500 \text{ nm}} (3,5 \cdot 10^{15} \lambda^2 - 2,25 \cdot 10^9 \lambda) d\lambda + \int_{500 \text{ nm}}^{700 \text{ nm}} (2,25 \cdot 10^9 \lambda - 1,5 \cdot 10^{15} \lambda^2) d\lambda \right]$$

$$= \frac{0,8q}{hc} \left[\left[3,5 \cdot 10^{15} \frac{\lambda^3}{3} - 2,25 \cdot 10^9 \frac{\lambda^2}{2} \right]_{300 \text{ nm}}^{500 \text{ nm}} + \left[2,25 \cdot 10^9 \frac{\lambda^2}{2} - 1,5 \cdot 10^{15} \frac{\lambda^3}{3} \right]_{500 \text{ nm}}^{700 \text{ nm}} \right]$$

$$= 41,9 \frac{\text{A}}{\text{m}^2} + 103,7 \frac{\text{A}}{\text{m}^2} = 14,6 \frac{\text{mA}}{\text{cm}^2}$$

f) Answer: $J_{sc} = 18,2 \text{ mA/cm}^2$

Using the same calculation steps as exercise (c) but now with the EQE of junction B, which

is 0,6 for $700 \text{ nm} < \lambda < 1200 \text{ nm}$, we get:

$$J_{sc} = \frac{q}{hc} \cdot 0,6 \int_{700 \text{ nm}}^{1200 \text{ nm}} \frac{hc}{\lambda} d\lambda = \frac{0,6q}{hc} \left[\int_{700 \text{ nm}}^{1200 \text{ nm}} (2,25 \cdot 10^9 \lambda - 1,5 \cdot 10^{15} \lambda^2) d\lambda \right]$$

$$= \frac{0,6q}{hc} \left[\frac{1}{2} \cdot 2,25 \cdot 10^9 \lambda^2 - \frac{1}{3} \cdot 1,5 \cdot 10^{15} \lambda^3 \right]_{700 \text{ nm}}^{1200 \text{ nm}} = 182,1 \frac{\text{A}}{\text{m}^2} = 18,2 \frac{\text{mA}}{\text{cm}^2}$$

2)

a)

Answer: $V_{oc} = 32,4 \text{ V}$

If all the cells are in series, the voltage will be added to get the open circuit voltage of the module.

$$V_{oc} = 0,6 \text{ V} \cdot 54 = 32,4 \text{ V}$$

b) Answer: $I_{sc} = 4 \text{ A}$

The short circuit current of a solar cell module with all the solar cells connected in series is the same as the solar cell with the lowest current. At STC, that means that the short circuit current will be 4 A.

c) Answer: $I_{sc} = 2 \text{ A}$

If there is a partial shading, the shaded cell will lower the current of the whole module to 2 A. That is why bypass diodes are introduced, to include an alternative path for the current of non-shaded cells when some of them are shaded.

3)

- a) ~~There are~~ Shading of individual cells results in a lower short circuit current I_{sc} since the illuminated area of the cell is smaller. The open circuit voltage V_{oc} under shading, with the assumption that $I_{sc} = I_0$, can be calculated as:

$$V_{oc} = \frac{k_B T}{q} \ln \left(\frac{I_{sc}}{I_0} + 1 \right)$$

This equation is also used to determine the saturation current I_0 of ~~that~~ all cells:

$$I_0 = \frac{I_{sc}}{e^{\frac{q V_{oc}}{k_B T} - 1}} \approx \frac{6 \text{ A}}{\exp \left(\frac{1,602 \cdot 10^{-19} \text{ C} \cdot 0,68 \text{ V}}{1,38 \cdot 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1} \cdot 300 \text{ K}} \right)} = 1,937 \cdot 10^{-14} \text{ A}$$

The I-V curves of the module are calculated with the ideal diode equation:

$$I(V) = I_0 \left[\exp \left(\frac{q V}{k_B T} \right) - 1 \right] - I_{sc}$$

There are 20 cells with which are shaded 30% of their area, this gives:

$$I_{sc} = 6 \text{ A} \cdot (1 - 0,3) = 4,2 \text{ A}$$

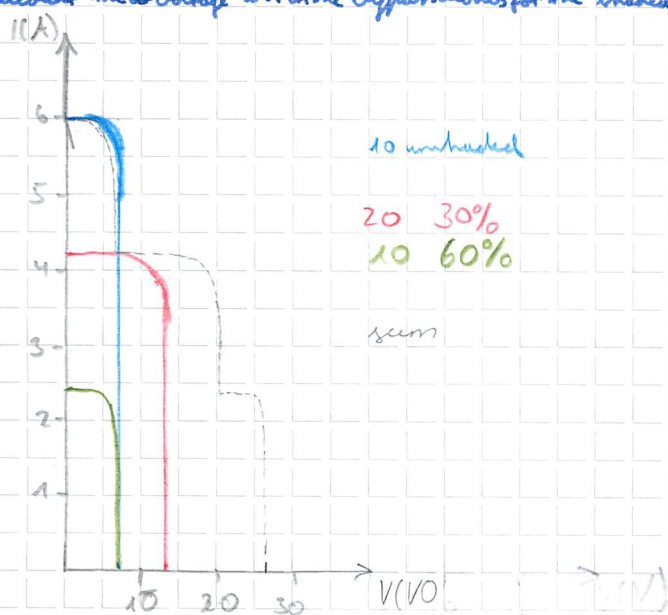
$$V_{oc} = \frac{1,38 \cdot 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1} \cdot 300 \text{ K}}{1,602 \cdot 10^{-19} \text{ C}} \cdot \ln \left(\frac{4,2 \text{ A}}{1,937 \cdot 10^{-14} \text{ A}} + 1 \right) = 0,671 \text{ V}$$

There are 10 cells which are shaded 60% of their area, this gives:

$$I_{sc} = 6 \text{ A} \cdot (1 - 0,6) = 2,4 \text{ A}$$

$$V_{oc} = \frac{1,38 \cdot 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1} \cdot 300 \text{ K}}{1,602 \cdot 10^{-19} \text{ C}} \cdot \ln \left(\frac{2,4 \text{ A}}{1,937 \cdot 10^{-14} \text{ A}} + 1 \right) = 0,656 \text{ V}$$

There are 10 cells which are not shaded, which have $V_{oc} = 0,68 \text{ V}$ and $I_{sc} = 6 \text{ A}$. The module I-V curve is obtained by summing the ~~current~~ current of the individual shaded and unshaded solar cell curves, taking into account the ~~voltage~~ voltage lost in the bypass diodes for the shaded solar cells.



3)

b) Answer: $P = 0,72 \text{ W}$

The total power lost in the bypass diodes is calculated by multiplying the constant forward voltage of 10 mV of each bypass diode with the current lost by shading:

$$20 \text{ cells, } 30\% \text{ shaded: } I = 6 \text{ A} - 2,2 \text{ A} = 1,8 \text{ A} \quad V = 10 \text{ mV} \cdot 20 = 0,2 \text{ V}$$

$$10 \text{ cells, } 60\% \text{ shaded: } I = 6 \text{ A} - 2,4 \text{ A} = 3,6 \text{ A} \quad V = 10 \text{ mV} \cdot 10 = 0,1 \text{ V}$$

$$P = 1,8 \text{ A} \cdot 0,2 \text{ V} + 3,6 \text{ A} \cdot 0,1 \text{ V} = 0,72 \text{ W}$$

c) Answer: 18 solar cells

In the worst case, when n solar cells are connected in series and only 1 cell is shaded, the voltage of the shaded cell V_s should be lower than the breakdown voltage $V_{\text{breakdown}}$. This voltage is calculated as:

$$V_s = V_F + (n-1) \cdot V_{OC} = 0,01 \text{ V} + (n-1) \cdot 0,68 \text{ V}$$

With V_F the constant forward voltage of the bypass diode.

$$V_s = n \cdot 0,68 \text{ V} - 0,67 \text{ V} < 12 \text{ V} \quad n \leq \frac{12 \text{ V} + 0,67 \text{ V}}{0,68 \text{ V}} = 18,6$$

Thus every 18 cells should have a bypass diode.

4) Answer: 6 steps

The MPPT should reach point H at 22 V starting from point A at 8 V , considering its two modes of operation.

1) In the first step, the MPPT moves the operating point by a coarse adjustment to point B, corresponding to a voltage of $8 \text{ V} + 5 \text{ V} = 13 \text{ V}$.

2) As the power increased in going from A to B, the next step is a coarse one. So the MPPT moves the operating point to C, corresponding to a voltage of $13 \text{ V} + 5 \text{ V} = 18 \text{ V}$.

3) As the power increased in going from B to C, the next step is a coarse one. So the MPPT moves the operating point to D, corresponding to a voltage of $18 \text{ V} + 5 \text{ V} = 23 \text{ V}$.

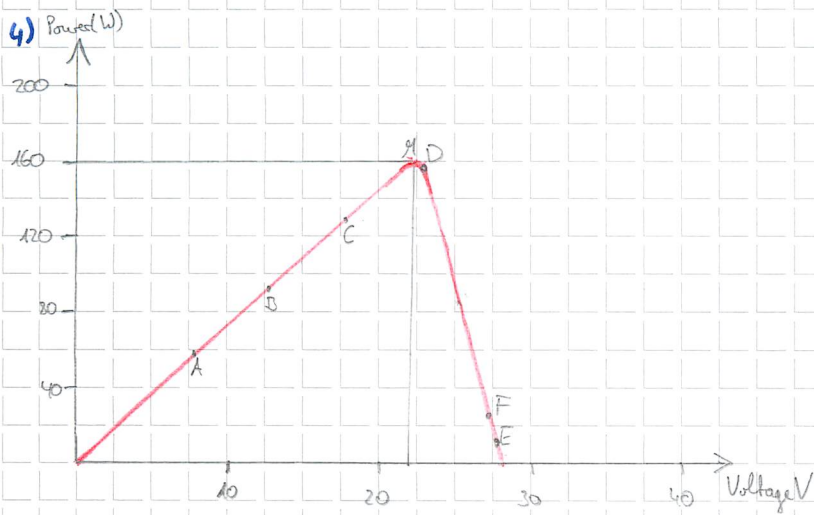
4) The operating point D has now crossed MPP, but the tracker doesn't know this yet. As the power increased in going from C to D, the next step is a coarse one. So the MPPT moves the operating point to E, corresponding to a voltage of $23 \text{ V} + 5 \text{ V} = 28 \text{ V}$.

5) As the move from D to E has drastically reduced the PV power, the MPPT reverses direction and performs a fine adjustment.

Thus, it moves the operating point to F, corresponding to a voltage of $28 \text{ V} - 1 \text{ V} = 27 \text{ V}$.

6) As the move from E to F has increased the power, the next step is a coarse one. So, the operating point moves to H, corresponding to a voltage of $27 \text{ V} - 5 \text{ V} = 22 \text{ V}$.

This is V_{MPP} , the MPPT has successfully tracked the MPP in 6 steps.



5)

a)

Answer: $E_{\text{bat}} = 1,2 \text{ kWh}$

The energy capacity of a battery is given by multiplying the rated battery voltage by the battery capacity C_{bat} :

$$E_{\text{bat}} = C_{\text{bat}} V = 100 \text{ Ah} \cdot 12 \text{ V} = 1200 \text{ Wh}$$

b) *

Answer: $I = 200 \text{ A}$

A C-rate of 1C (dis)charges the entire battery in 1 hour, and therefore corresponds to a current of $I = 100 \text{ A}$. Consequently, a C-rate of 2C (dis)charges the entire battery in 0,5 hour, and therefore corresponds a current of $I = 200 \text{ A}$

c)

Answer: 18 minutes

A C-rate of 2C (dis)charges the entire battery in 0,5 hour. So 60% of the battery is charged in $60\% \cdot 0,5 \text{ hour} = 18 \text{ minutes}$

d)

Answer: $\eta_{\text{bat}} = 81\%$

The battery efficiency is defined as the product of voltaic and coulombic efficiency: $\eta_{\text{bat}} = \eta_V \cdot \eta_C = 90\% \cdot 90\% = 81\%$